RP-003-001544 **Seat No. _______**

B. Sc. (Sem. V) (CBCS) Examination

February - 2019

S-503 : Statistical Inference

(Old Course)

Faculty Code : 003

Subject Code : 001544

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) Q. 1 carry 20 marks.

(ii) Q. 2 and Q. 3 carry 25 marks each.

(iii) Student can use their own scientific calculator.

1 Filling the blanks and short questions : (Each 1 mark) **20**

(1) Estimation is possible only in case of a ______.

(2) A sample constant representing a population parameter is known as ______.

(3) If T_n is an estimator of a parametric function $\tau(\theta)$, the mean square error of T_n is equal to ______.

(4) $\sum \frac{x_i}{n}$ for $i = 1, 2, 3, ..., n$ is a ______ estimator of population mean.

(5) For mean square error to be minimum, bias should be _____.

(6) An estimator of $v_\theta(T_n)$ which attains lower bound for all θ is known as _______.

(7) If $S = s(X_1, X_2, X_3, ..., X_n)$ is a sufficient statistic for θ of density $f(x;\theta)$ and $f(x_i;\theta)$ for $i = 1, 2, 3, ..., n$ can be factorised as $g(s,\theta)\,h(x)$, then $s(X_1, X_2, X_3, ..., X_n)$ is a ____.

(8) If a random sample $x_1, x_2, x_3, ..., x_n$ is drawn from a population $N(\mu, \sigma^2)$, the maximum likelihood estimate of σ^2 is _____.

(9) For a Gama (x, α, λ) distribution with λ known, the maximum likelihood estimate of α is _____.

(10) Maximum likelihood estimate of the parameter θ of the distribution $f(x,\theta) = \frac{1}{2} e^{-|x-\theta|}$ is _____.

(11) ______ is an unbiased estimator of p^2 in Binomial distribution.

(12) The estimate of the parameter λ of the exponential distribution $\lambda e^{-\lambda x}$ by the method of moments is _____.

(13) For a rectangular distribution $\frac{1}{(\beta - \alpha)}$, the maximum likelihood estimates of α and β are ________ and ________ respectively.

(14) If $x_1, x_2, x_3, ..., x_n$ is a random sample from an infinite population and s^2 is defined as $\frac{\sum (x_i - \bar{x})^2}{n}$, $\frac{n}{n-1}S^2$ is an _______ estimator of population variance σ^2.

(15) Let there be a sample of size n from a normal population with mean μ and variance σ^2. The efficiency of median relative to the mean is ______.

(16) Minimum Chi-square estimators are not necessarily ________.

(17) If a function $f(t)$ of the sufficient statistics $T = t(x_1, x_2, x_3, ..., x_n)$ is unbiased for $\tau(\theta)$ and is also unique, this is the _________.

(18) If sufficient estimator exists, it is function of the _______.

(19) Sample mean is an ________ and _________ estiamte of population mean.

(20) If T_1 and T_2 are two MVU estimator for $T(\theta)$, then ________.

2 (a) Write the answer any **three** : (Each 2 marks) **6**

(1) Define unbiasedness.

(2) Define efficiency.

(3) Define complete family of distribution.

(4) Define uniformly most powerful test (UMP test).

(5) Define ASN function of SPRT.

(6) Find the Cramer Rao lower bound of variance of unbiased estimator of parameter of the probability distribution $f(x, \theta) = \theta e^{-\theta x}$.

(b) Write the answer any **three** : (Each 3 marks) **9**

(1) Obtain unbiased estimator of $\frac{kq}{p}$ of negative binomial distribution.

(2) $\frac{\overline{x}}{n}$ is a consistent estimator of p for binomial distribution.

(3) Obtain MVUE of parameter θ for Poisson distribution. Also obtain its variance.

(4) Obtain estimator of θ by method of moments in the following distribution.

$f(x;\theta) = \theta e^{-\theta x}$; where $0 \le x \le \infty$.

(5) Obtain operating characteristics (OC) function of SPRT.

(6) Give a random sample $x_1, x_2, x_3, \ldots\ldots, x_n$ from distribution with p.d.f. $f(x;\theta) = \frac{1}{\theta}$; $0 \le x \le \theta$. Obtain power of the test for testing $H_0 : \theta = 1.5$ against $H_1 : \theta = 2.5$ where $c = \{x; x \ge 0.8\}$.

(c) Write the answer any **two** : (each 5 marks) **10**

(1) State Crammer-Rao inequality and prove it.

(2) Estimate α and β in the case of Gamma distribution by the method of moments.

$$f(x;\alpha,\beta) = \frac{\alpha^{\beta}}{\Gamma\beta} e^{-\alpha x} x^{\beta-1}; x \ge 0, \alpha \ge 0$$

(3) Obtain OC function for SPRT of Binomial distribution for testing $H_0 : p = p_0$ against $H_1 : p = p_1 (> p_0)$.

(4) Give a random sample $x_1, x_2, x_3, \ldots\ldots, x_n$ from distribution with p.d.f.

$$f(x;\theta) = \theta e^{-\theta x};\ 0 \le x \le \infty,\ \theta > 0$$

Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

(5) Obtain likelihood ration test :

Let $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ random sample taken from $N(\mu, \sigma^2)$. To test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \ne \sigma_0^2$.

3 (a) Write the answer any **three** : (any 2 marks) **6**

(1) Define consistency.

(2) Define sufficiency.

(3) Define minimum variance bound estimator (MVBE).

(4) Define most powerful test (MP test).

(5) Obtain likelihood function of Laplace distribution.

(6) Obtain an unbiased estimator of θ by for the following distribution.

$$f(x;\theta) = \frac{1}{\theta};\ 0 \le x < \theta$$

(b) Write the answer any **three** : (each 3 marks) **9**

(1) Let $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ be random sample taken from $N(\mu, \sigma^2)$ then find sufficient estimator of μ and σ^2.

(2) Obtain an unbiased estimator of population mean of χ^2 distribution.

(3) Prove that $E\left(\frac{\partial \log L}{\partial \theta}\right)^2 = -E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right)$.

(4) If A is more efficience then B then prove that $Var(A) + Var(B - A) = Var(B)$.

(5) Use the Neyman Pearson lemma to obtain the best critical region for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$ in the case of Poisson distribution with parameter λ.

(6) Let p be the probability that coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 head are obtained. Find the probability of type-I error, type-II error and power of test.

(c) Write the answer any **two** : (each 5 marks) **10**

(1) State Neyman-Pearson lemma and prove it.

(2) Obtain MVBE of σ^2 for normal distribution $(0, \sigma^2)$.

(3) If T_1 and T_2 be two unbiased estimator of θ with variance σ_1^2, σ_2^2 and correlation ρ, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination ?

(4) For the double Poisson distribution

$$P(X=x)=\frac{1}{2}\frac{e^{-m_1}m_1^x}{x!}+\frac{1}{2}\frac{e^{-m_2}m_2^x}{x!};\ 0,1,2,......$$

Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1' \pm \sqrt{\mu_2' - \mu_1' - \left(\mu_1'\right)^2}$.

(5) Construct SPRT of Poisson distribution for testing $H_0:\lambda=\lambda_0$ against $H_1:\lambda=\lambda_1(>\lambda_0)$. Also obtain OC function of SPRT.